CONJECTURE ACTIVITIES

for comprehending statistics terms

through speculations on the functions of imaginary spectrometers

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Introduction

The purpose of this study was to describe students' problem solving performance when they make conjectures to comprehend three statistics terms. Teachers are key figures in changing the ways in which mathematics is taught and learned in schools. Mathematics teachers are supposed to design meaningful tasks to motivate students' interest and to enhance students' communication and reasoning. Within various contexts, if meaningful tasks are designed for students to work on, then students should benefit more from those contexts of problem solving. For example, in this unit for learning three statistics terms, i.e., median, mode and range, the authors provided opportunities for students to conjecture, verify, and modify their rules rather than directly telling them the rules to find those three statistics terms. Such a learning process might result in better student performance.

The National Council of Teachers Mathematics (NCTM, 2000) stressed that middle grade students should have the opportunities to make conjectures and design experiments or surveys to collect relevant data. In the middle grades, students should learn to use the median, the mode and the range, to describe distribution of a data set. Students finally need to understand that the median indicates the 'middle' of a data set, the mode highlights the most prevalent sample of a data set, and the 'range' is one measure of the spread of a data set. Students often fail to comprehend the meaning of those statistics terms. Thus, mathematics teachers have an important role in providing experiences for students to comprehend statistics terms.

Inductive inference is expected to appear during the process of conjecturing. An induction is an inference of a generalised conclusion from

particular instances. Inductive processes will produce a net increase in knowledge, yet must be severely constrained if they are to produce plausible conclusions. What is more, an induction is risky in the sense that it may not be true, even if the premises are true (Bisanz, Bisanz & Koepan 1994).

In addition, the form of activity, the kind of thinking required, and the way in which students are led to explore the particular content all contribute to the kind of learning opportunity afforded by the task. To capitalize on this opportunity, teachers should deliberately select tasks that provide them with windows into the students' thinking.

Teachers should consider what they know about their particular students not only from a mathematics learning perspective, but from psychological, cultural, and sociological ones as well. Sensitivity to the diversity of students' backgrounds and experience is crucial in selecting worthwhile tasks.

The purpose of this article is to introduce the design of conjecture activities, to describe students' dialogues in activities and to discuss the implications from these activities.

The design of conjecture activities

A spectrometer is an instrument used to disperse radiant energy or particles into a spectrum and measure certain properties such as wavelength, mass, energy, or index of refraction. To conjecture, verify, and modify the relationships between data and answers, three statistics terms were first temporarily replaced by three imaginary spectrometers in the conjecture activities. We label these as 'spectrometers' because the spectrometers have the functions of transmission, scattering, concentration and relocating. The spectrometers were not real instruments, but the name is being used metaphorically. This design has three stages. First, the source data and the target answers were shown (Figures 2, 3, and 4) to students. The functions of those imaginary spectrometers were conjectured by students. Second, based on the analyses of the relationships between data and results, students conjectured, discussed and negotiated the functions of the three spectrometers. Finally, students were asked to do a matching activity to connect three statistics terms and three spectrometers. In addition, students were asked to find the statistical definitions of the three based on their functions.

Figure 1. Looking for the pattern from a conjecture activity.

The conjecture activities

Ms Ho is an expert teacher. Her fifth grader's classroom had finished the unit of arithmetic average. She found some students could find the average but failed to understand the meaning of average. She then tried to utilise conjecture strategies to teach the other statistics terms. There are three stages during the conjecture activities.

Stage 1: Commencement

Ms Ho said, "In our class, some student assistants have special responsibilities to help the teachers. One student is in charge of the discipline in the classroom when the teachers are absent, another student is in charge of cleaning the classroom, and another student looks after sport equipment. Each assistant has his/her own function."

Ms Ho introduced three spectrometers by drawing an analogy with the duty of assistants in the classroom. As every student assistant has his/her own function, each of three spectrometers performs its own functions. Let us find their functions from the following conjecture activities.

Stage 2: The dialogues in conjecture activities

The dialogues of one group which had a lot of debates and refutation has been recorded and analysed. The symbols S1, S2, represent different students in this group. The functions and names of three imaginary spectrometers which represent median, mode, and range will be conjectured by students. As the author expected, students could comprehend statistics rules from conjecturing activities and inductive speculation.

Episode 1: No. 1 Spectrometer for Learning Median

S1: Finding the common function from those data seems difficult. It is similar to guessing the security password of a software package.

S2: Let start from this set (1, 3, 5, 7, 9)! (1, 3, 5, 7, 9). The answer is 5. Is 5 obtained by deleting from both sides 1 and 9, then 3 and 7?

S3: It seems right. Let us check another set of data.

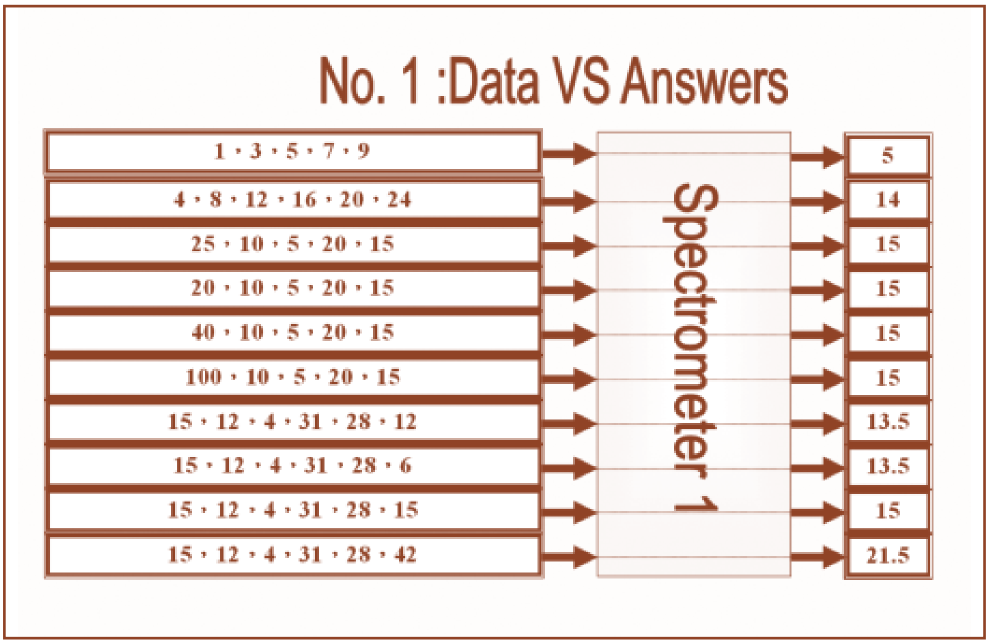

Figure 2. Data and answers of Spectrometer 1.

S2: Look at this set, (4, 8, 12, 16, 20, 24). If we delete data from both sides of this set, 4 and 24 then 8 and 20, it still has two numbers 12 and 16. Which one is correct? The answer is 14, neither 12 nor 16.

S4: Is 14 the 'most' middle number in the set?

S3: Yes, it might be possible. Let us use this strategy to check the third set (25,10, 5, 20, 15). By deleting from both sides twice, the answer is 15, rather than 5. Why?

S4: 15 is also one of the numbers in that set. Is it not?

S3: Let us rearrange the order of this set by the sequence from small to large. We get the new set, (5, 10, 15, 20, 25).

S1: Oh, I got it! The function of No. 1 is to find the middle number of the data sequence.

S4: It's possible. Let us check this conjecture using the other data sets.

T: Do you think that the function of No. 1 spectrometer is to cancel both numbers from the largest and smallest ones and what remains (the most middle number) is the answer?

S3: Yes!

T: After deleting by pairs, did you finally find a single number was left in one set of data?

S3: It depends! When there are even numbers in a set, the final results deleted by pairs are two numbers rather than one.

T: Then, how do you get the exact one?

S3: We can use the 'average' strategy to find the most middle one for the two numbers!

T: Is it true for all cases?

S4: Yes, give the 'average' for the two numbers from the even sequence data! Then, it works for every set.

From the process of conjecturing, students found the pattern once the set had been rearranged by order from smallest to largest. Students are pleased to say so during the process of conjecturing. They then immediately test another set having even numbers of data to confirm their conjecturing.

Ms Ho then asked students to name the spectrometer. Students gave different names to No. 1 spectrometer, such as the middle number, the centre number, the balance number, Libra, and Impartial God. The first three names are similar to mathematics terms. The last two names seem to reflect the role of helpers. The reason they chose Libra is that it stands on the middle position on the pivot of a balance. The explanation for why they chose the 'Impartial God' is that it stands on the centre of the data sequence and is impartial to any side.

Episode 2: No. 2 Spectrometer for Learning Mode

S1: Here is another spectrometer! Its function may be different from No1. This number sequence, (2, 2, 4, 5, 9, 2, 7), passed through No. 2 and got the result 2. What is this pattern?

Students tried the smallest number, the common factors, and the least common factor. Then Ms Ho suggested students consider several sets of data and answers together.

S2: Let's see! (2, 2, 4, 5, 9, 7, 2) turned to 2, (3, 8, 8, 5, 8, 6, 13, 8, 3) turned to 8, (10, 15, 5, 10, 10, 20, 10, 5) turned to 10. It seems they have repeated numbers in each set.

S3: Is possible to find the 'repeated number' as the result?

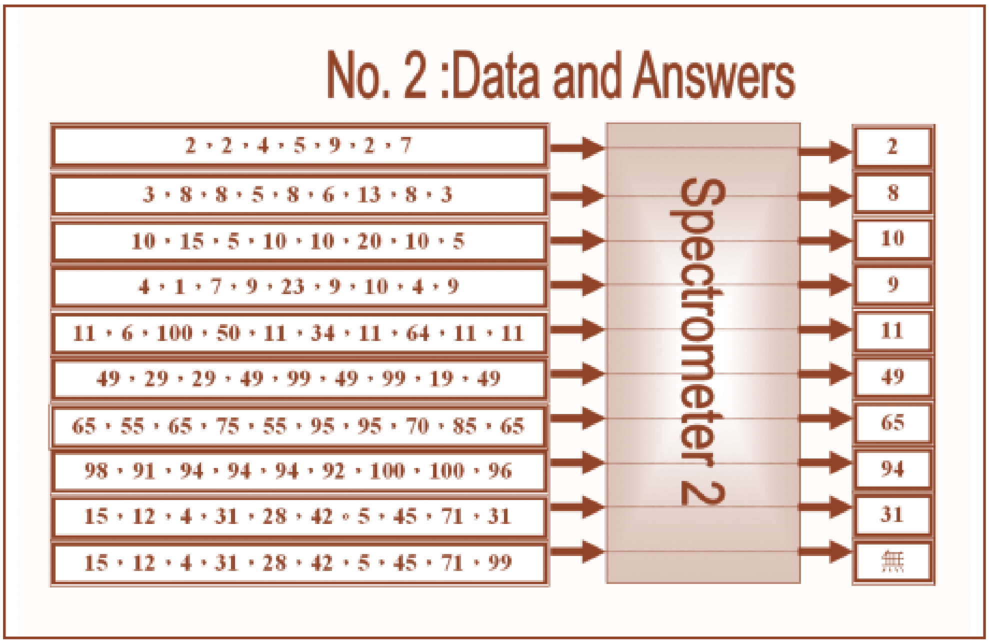

Figure 3. Data and answers of Spectrometer 2.

S1: Let's check the other sets.
S4: Yes, No. 2 spectrometer is to find the most prevalent numbers!

After Ms Ho encouraged students to take several sets together, they thus found the pattern and used their own words to stand for the repeated numbers. They name No. 2 spectrometer "the copy devil," "the identical number," "multiple fetus" and "most frequent number." Finally, they agreed the function of No. 2 is to find the most prevalent number.

Episode 3: No. 3 Spectrometer for Learning Range
Students utilised the previous strategies to guess No. 3. In addition, most of the ranges, being the distances from the smallest one to the largest one; do not appear in the data set. Students tried using the middle number, 5,

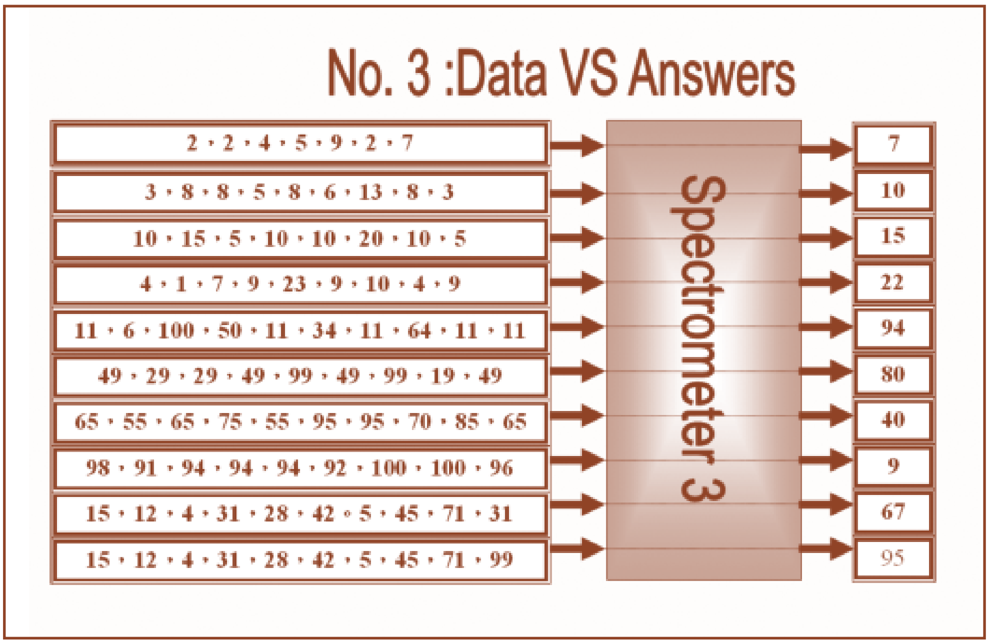

Figure 4. Data and answers of Spectrometer 3.

added 2, then got 7; or the repeated number subtracted by the smallest number. Ms Ho thus suggests that they rearrange the data sequence from the smallest to the largest and look for the pattern.

S4: I got it! It's that the largest number minus the small one. Let's check the other sets.

After students rearranged the data sequence, they got the answers. In fact, the titles of No. 3 given by students are far from the term 'range'. However, a suitable term, the difference calculator, has been represented during the process of identifying function. In the long run, students discovered that the No. 3 spectrometer finds the span between the largest and the smallest data entries.

If students have experiences in locating numbers on number lines and identifying that the difference is the distance between any two points on the number line, they might more quickly identify the functions by their own words and might use the closer term of 'range' during the conjecture activity. Students finally find the functions of three imaginary spectrometers from the process of conjecturing, and induction.

Stage 3: A matching-up activity and giving meaning for terms

Students can understand basic concepts of three terms from the process of giving names. After they experienced the activities of conjecturing and naming, students realised the role of three statistic terms. During the third stage, the authors asked students to carry out a matching up activity between three statistics terms and three imaginary spectrometers. Students quickly connect the term 'median' to No. 1 spectrometer because median in Chinese possesses the meaning of 'centre position'. They also easily assigned the term of 'mode' to No. 2 spectrometer due to the term's meaning in Chinese indicating plenty or abundance. English speaking children would have difficulty in catching the meaning of these terms. Although they were less confident in naming No. 3 spectrometer than in naming the other two, the corresponding matching up activity was still easy for them to solve because after they chose the two terms of No. 1 and No. 2 spectrometers, the third term, range, was naturally matched to No. 3.

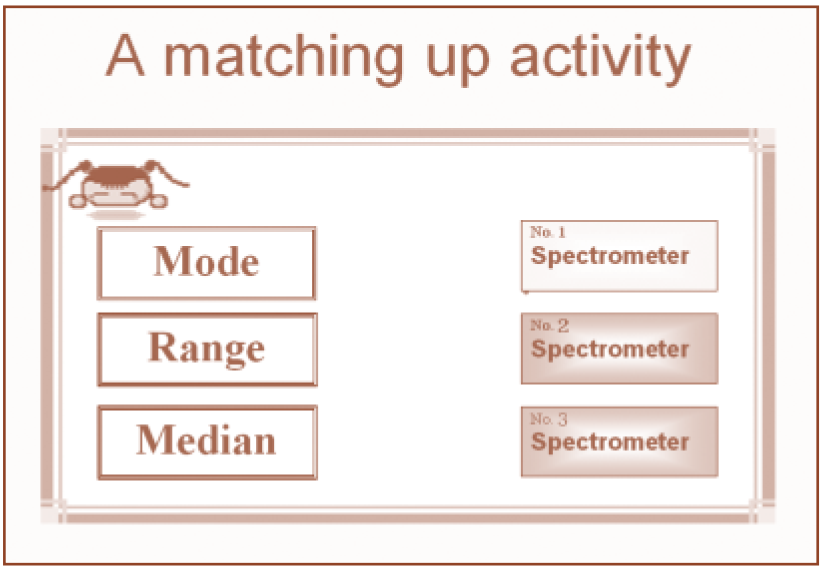

Figure 5. A matching up activity.

Figure 6. Naming activities for three spectrometers.

Discussion and reflections

As Lobato, Clarke and Ellis (2005) point out, just telling students how to deal with algorithms is likely to foreclose the discussion without contributing much to their understanding of the concepts. Instead of telling the rules, then asking students to copy and practice the rules of statistics terms, the teacher conducted the conjecturing, discussion, and induction activities to introduce the statistics terms. In addition, when students encountered difficulty, the teacher posed questions such as: “Why don’t you rearrange the data set from the smallest to the largest one?” or “Why don’t you consider several data sets together.” These questions really help students to scaffold their learning. Under this atmosphere of scaffolded teaching, students were aware of what they were doing, and frequently adjusted their strategies as they solved problems

The process of conjecturing hinges on being able to recognise a pattern or analogy, in other words, on being able to make a generalisation. As Bisanz, Bisanz, and Korpan (1994) stressed, some conclusions induced from conjecturing might be wrong and those temporary conclusions have to be checked by the other data sets. During the process of the conjecturing activities in stage 2, students met with many difficulties, such as, “Why did the hypothesis fit the former data set and not the latter data set?” Students are encouraged thus to make new hypotheses to fit the whole data. Giving up the original hypotheses and setting up a new hypothesis for survival is a process of adaptation. In addition, getting another data set to fit the new hypothesis is a process of accommodation. During the process of conjecture, students were engaged in the process of adaptation and accommodation; hence they develop basic concepts of those statistics terms.

In Episode 2, students used the “smallest number,” “common factors,” and “least common factor.” In Episode 3, a student found the pattern may be from the smallest number subtract the repeated number. As Bisanz, Bisanz, and Koepan (1994) emphasised, inductive processes must be severely constrained if they are to produce plausible conclusions. From the above episodes, the temporarily wrong conclusions are generated when students make inferences from very few data sets.

As Mason, Burton, and Stacey (1985) claimed, generalisation reasoning

involves focusing on certain aspects common to many examples, and ignoring other aspects; the process of generalizing is that of moving from a few instances to making informed guesses about a wide class of cases. To generalise their findings, students engage in the cyclic process of making a hypothesis, checking the answer and generating another hypothesis during the process of generalisation. This kind of mathematics activity resembles that of scientific investigation (Bisanz, Bisanz & Korpan, 1994).

As NCTM (1991) stressed, mathematics instruction needs to be orientated away from an emphasis on mechanistic answer-finding, and towards conjecturing, and problem solving. This study showed that students could infer statistics rules from inductive speculation and the functions of three spectrometers from the process of conjecturing, verifying and modifying. In addition, students could provide intuitive terms for the functions of the three spectrometers and they then perhaps realise the basic concepts of the three formal statistic terms from the process of giving names.

If students found the process of conjecturing problematic, how did they proceed? What information did they use? What misconceptions did they have? Students' problematic experiences can serve as a springboard to pose new problems for students to improve their mathematical ability. From this article we found that conjecture activities are beneficial for initiating statistics lessons. The conjecture activities can stimulate the mechanism of adaptation and accommodation. Given these findings, the question we must ask ourselves is: what kinds of lessons are suitable for utilising conjecture activities?

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